

Constraints on the extensions to the base Λ CDM model from BICEP2, Planck and WMAP

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Recently Background Imaging of Cosmic Extragalactic Polarization (B2) discovered the relic gravitational waves at 7.0σ confidence level. However, the other cosmic microwave background (CMB) data, for example Planck data released in 2013 (P13), prefer a much smaller amplitude of the primordial gravitational waves spectrum if a power-law spectrum of adiabatic scalar perturbations is assumed in the six-parameter Λ CDM cosmology. In this paper, we explore whether the w CDM model and the running spectral index can relax the tension between B2 and other CMB data. In particular, we find that a positive running of running of spectral index is preferred at 1.7σ level from the combination of B2, P13 and WMAP Polarization data.

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I. INTRODUCTION

In the early of 2013, Planck (P13) [1] released its data which precisely measured the temperature anisotropies of cosmic microwave background (CMB), and claimed that it strongly supports the standard spatially-flat six-parameter Λ CDM cosmology with a power-law spectrum of adiabatic scalar perturbations. Actually the relic gravitational waves could also make contributions to the temperature and polarization power spectra in the CMB [2–6]. Combining Wilkinson Microwave Anisotropy Probe (WMAP) 9-year data [7] with Baryon Acoustic Oscillation (BAO) [8], H_0 prior from Hubble Space Telescope (HST) [9] and other highL CMB data, including Atacama Cosmology Telescope (ACT) [10] and South Pole Telescope (SPT) [11], we obtained the constraint on the primordial gravitational waves before Planck as follows

$$r_{0.002} < 0.12 \quad (1)$$

at 95% CL in [12], where $r_{0.002}$ is the tensor-to-scalar ratio at the pivot scale $k_p = 0.002 \text{ Mpc}^{-1}$ and a power-law spectrum of the primordial scalar perturbations is also assumed. A similar result was reported by Planck combining with WMAP polarization (WP) data and other highL CMB data, namely

$$r_{0.002} < 0.11 \quad (2)$$

at 95% CL in [1]. In this paper, we shall fix the pivot scale as $k_p = 0.002 \text{ Mpc}^{-1}$.

Considering that the primordial gravitational waves only make contributions to CMB power spectra at the

very large scales, we fixed the background parameters as their best-fit values from Planck, and then run the CosmoMC to figure out the amplitude of adiabatic scalar perturbations, spectral index and the tensor-to-scalar ratio by only using the low-multipole Planck TT [1] and WMAP TE (WP) data [7]. We found $r > 0$ at more than 68% confidence level with maximum likelihood at around $r \sim 0.2$ [13]. Our new result confirmed the previous one in [14] where WMAP 7-year data were utilized. Recently Background Imaging of Cosmic Extragalactic Polarization (B2) [15] discovered the relic gravitational waves with the tensor-to-scalar ratio

$$r = 0.20^{+0.07}_{-0.05}, \quad (3)$$

and $r = 0$ is disfavored at 7.0σ . Using B2 only or the combination of B2, P13 and WP, the tilt n_t of relic gravitational waves spectrum is constrained to be around zero and $n_t = 2$ is ruled out at more than 5σ confidence level in [16, 17] which strongly indicates that inflation [18–20] really happened in the early Universe.

In this paper we hope to get a better understanding about the physics in our Universe through a more careful investigation of the datasets. Comparing (3) to (1) and (2), we see that there is a moderately strong tension between B2 and other CMB data in the base six-parameter Λ CDM+tensor cosmology. If all of these CMB datasets are trustable, it strongly implies that our Universe is much more complicated than what we expected before. In order to reconcile the tension on constraining the primordial gravitational waves between P13 and B2, we need to go beyond the Λ CDM+tensor model. There are several well-motivated extensions to the Λ CDM+tensor model which might relax such an inconsistency.

i) More complicated physics in the early universe can be involved. Here we consider that the spectrum of adiabatic scalar perturbations departs from a pure power-law form, and the running of spectral index ($dn_s/d \ln k$)

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and the running of running ($d^2 n_s / d \ln k^2$) are taken into account. Or the spatial curvature (Ω_k) of our Universe deviates from exact flatness.

ii) We can consider more complicated physics about neutrino and relativistic components by relaxing the total mass of active neutrinos ($\sum m_\nu$), or the number of relativistic species (N_{eff}).

iii) The abundance of light elements, for example $Y_P \equiv 4n_{\text{He}}/n_b$ for helium-4, is taken as a free parameter.

iv) The dark energy is not a cosmological constant and its equation-of-state (EOS) parameter $w \equiv p_{\text{de}}/\rho_{\text{de}}$ is regarded as a free parameter.

An almost comprehensive investigation has been given by Lewis in [21] where the combination of B2 and P13 was considered. In this paper we will adopt B2, P13 and WP to explore two extensions: one is to relax the dark energy model from cosmological constant to one with a constant EOS parameter $w = p_{\text{de}}/\rho_{\text{de}}$; the other is to take into account the running of spectral index and the running of running. Here we fix the consistency relation to be $n_t = -r/8$.

II. IMPLICATIONS FOR COSMOLOGY FROM BICEP, PLANCK AND WMAP

In this section we will use the CosmoMC [22] to work out the constraints on the cosmological parameters in different cosmological models from several different combinations of datasets respectively. Our results are summarized in Tables I, II and III.

A. w CDM model

In this subsection, we extend the dark energy model from the cosmological constant to the dark energy with constant EOS parameter w . As we know, there are also several tensions between P13 and some local cosmological observations, including the H_0 prior from HST [9] and Supernova Legacy Survey (SNLS) samples [23]. For example, P13 prefers a larger matter density today compared to SNLS and a smaller Hubble constant compared to the H_0 prior from HST. However these two tensions can be significantly relaxed in the w CDM model [24] where the dark energy is preferred to be phantom-like, namely $w = -1.16 \pm 0.06$ from the combination of P13+WP+BAO+SNLS+HST.

Here we also wonder whether the dark energy EOS can help to relax the tension on the tensor-to-scalar ratio between P13 and B2. We constrain the cosmological parameters in the w CDM+r model by adopting the combinations of P13+WP and B2+P13+WP, respectively. See the results in Table I and Fig. 1. We find that the constraint on r is given by

$$r_{0.002} < 0.16 \quad (4)$$

w CDM+r	B2+P13+WP	
parameters	best fit	68% limits
$\Omega_b h^2$	0.02222	$0.02210^{+0.00058}_{-0.00063}$
$\Omega_c h^2$	0.1161	$0.1172^{+0.0029}_{-0.0028}$
$100\theta_{\text{MC}}$	1.04190	$1.04163^{+0.00067}_{-0.00068}$
τ	0.1001	$0.0888^{+0.0145}_{-0.0186}$
$\ln(10^{10} A_s)$	3.200	3.186 ± 0.034
n_s	0.9690	$0.9669^{+0.0140}_{-0.0163}$
$r_{0.002}$	0.16	$0.16^{+0.04}_{-0.05}$
w	-1.70	$-1.54^{+0.17}_{-0.32}$

TABLE I: Constraints on the cosmological parameters in the w CDM+r model.

at 3σ confidence level from P13+WP. There is still a more than 3σ tension on r between P13+WP and B2 in the w CDM+r model. Therefore relaxing the dark energy model cannot reconcile the tension on r between P13 and B2. Due to such a tension, some exotic results appear. For example, the constraint on the dark energy EOS parameter becomes $w = -1.54^{+0.17}_{-0.32}$ in Table. I. A similar constraint on w from B2+P13 is $w = -1.55^{+0.18}_{-0.34}$ in [25].

B. Running spectral index

In this subsection, we extend the six-parameter base Λ CDM+r model to the Λ CDM+nrun+r and Λ CDM+nrun+nrunrun+r models respectively, where nrun and nrunrun denote the running of spectral index ($dn_s/d \ln k$) and the running of running ($d^2 n_s / d \ln k^2$). In this case the amplitude of scalar perturbation spectrum is parameterized by

$$P_s(k) = A_s \left(\frac{k}{k_p} \right)^{n_s - 1 + \frac{1}{2} \frac{dn_s}{d \ln k} \ln \frac{k}{k_p} + \frac{1}{6} \frac{d^2 n_s}{d \ln k^2} \ln^2 \frac{k}{k_p}}. \quad (5)$$

In [12], the constraint on the tensor-to-scalar ratio from the combination of WMAP+ACT+SPT+BAO+HST is relaxed to be

$$r_{0.002} < 0.42 \quad (6)$$

at 95% CL if the running of spectral index is considered, and

$$r_{0.002} < 0.53 \quad (7)$$

at 95% CL if both the running and running of running are taken into account. In [1], the constraint on the tensor-to-scalar ratio is relaxed to be

$$r_{0.002} < 0.26 \quad (8)$$

at 95% CL from the combination of P13+WP+ACT+SPT if the running of spectral index is considered. We see that the constraint on r can

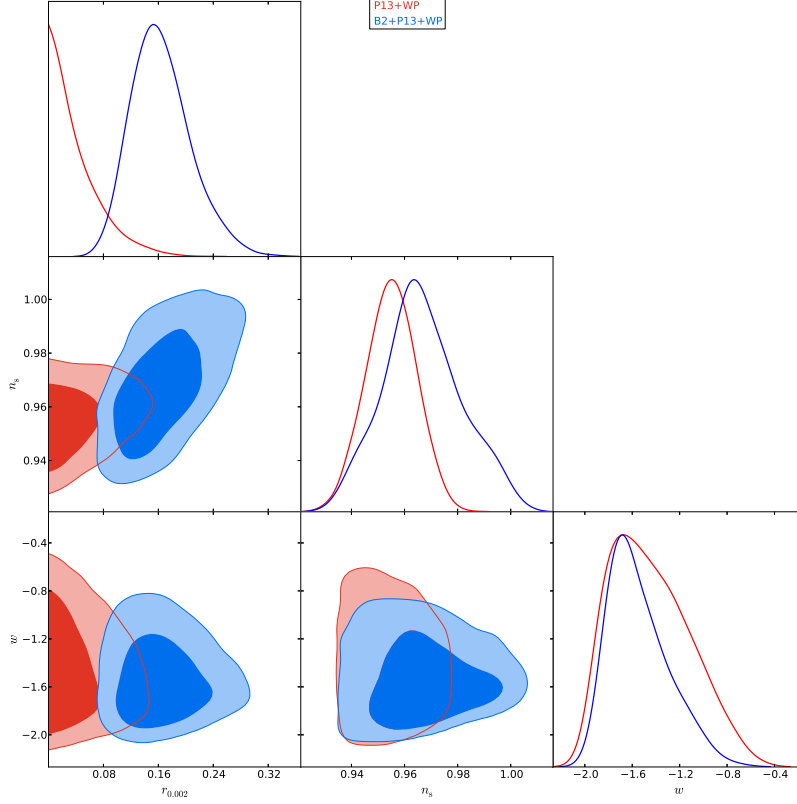


FIG. 1: The constraint contours on r , n_s and w from the combinations of P13+WP and B2+P13+WP in the w CDM+r model.

be significantly loosen to be consistent with B2 in the model with running spectral index.

First of all, we combine B2 with P13 and WP to constrain the cosmological parameters in the Λ CDM+nrun+r cosmology. Our results are given in Table II and Fig. 2. We see that a blue tilted scalar power

Λ CDM+nrun+r	B2+P13+WP	
parameters	Best fit	68% limits
$\Omega_b h^2$	0.02229	$0.02246^{+0.00030}_{-0.00032}$
$\Omega_c h^2$	0.1187	$0.1173^{+0.0022}_{-0.0021}$
$100\theta_{MC}$	1.04154	$1.04162^{+0.00061}_{-0.00062}$
τ	0.0991	$0.1011^{+0.0137}_{-0.0161}$
$\ln(10^{10} A_s)$	3.117	$3.098^{+0.045}_{-0.041}$
n_s	1.0336	$1.0447^{+0.0295}_{-0.0297}$
$dn_s/d \ln k$	-0.0228	-0.0253 ± 0.0093
$r_{0.002}$	0.18	$0.22^{+0.04}_{-0.07}$

TABLE II: Constraints on the cosmological parameters in the Λ CDM+nrun+tensor model.

spectrum at $k_p = 0.002 \text{ Mpc}^{-1}$ is preferred at 1.5σ level, and a negative running of spectral index is favored at around 2.7σ level. Combining with P13, WP and other highL CMB data, B2 implies $dn_s/d \ln k = -0.028 \pm 0.009$ [15]. In [21], the combination of B2+P13 gives a constraint $dn_s/d \ln k = -0.028 \pm 0.020$. In [25], $dn_s/d \ln k =$

-0.0281 ± 0.0099 from B2+P13+BAO+SN. See the analysis in [26, 27] as well. Our results are consistent with all of these previous results.

Since a negative running of spectral index is preferred at high confidence level, we wonder whether the higher order terms in the parametrization of scalar perturbation spectrum are required. Here we further extend the previous model to the Λ CDM+nrun+nrunrun+r model. The results show up in Table. III and Fig. 3. Compared

Λ CDM+nrun+nrunrun+r	B2+P13+WP	
parameters	Best fit	68% limits
$\Omega_b h^2$	0.02221	0.02217 ± 0.00035
$\Omega_c h^2$	0.1203	0.1184 ± 0.0023
$100\theta_{MC}$	1.04145	$1.04142^{+0.00064}_{-0.00063}$
τ	0.0983	$0.1054^{+0.0142}_{-0.0168}$
$\ln(10^{10} A_s)$	3.047	$3.063^{+0.066}_{-0.050}$
n_s	1.1656	$1.1344^{+0.0612}_{-0.0608}$
$dn_s/d \ln k$	-0.139	$-0.108^{+0.049}_{-0.048}$
$d^2 n_s/d \ln k^2$	0.045	$0.033^{+0.018}_{-0.019}$
$r_{0.002}$	0.22	$0.24^{+0.05}_{-0.07}$

TABLE III: Constraints on the cosmological parameters in the Λ CDM+nrun+nrunrun+tensor model.

to the previous model with only the running of spectral index, $\Delta\chi^2 = 9852.70 - 9855.82 = -3.12$ which indi-

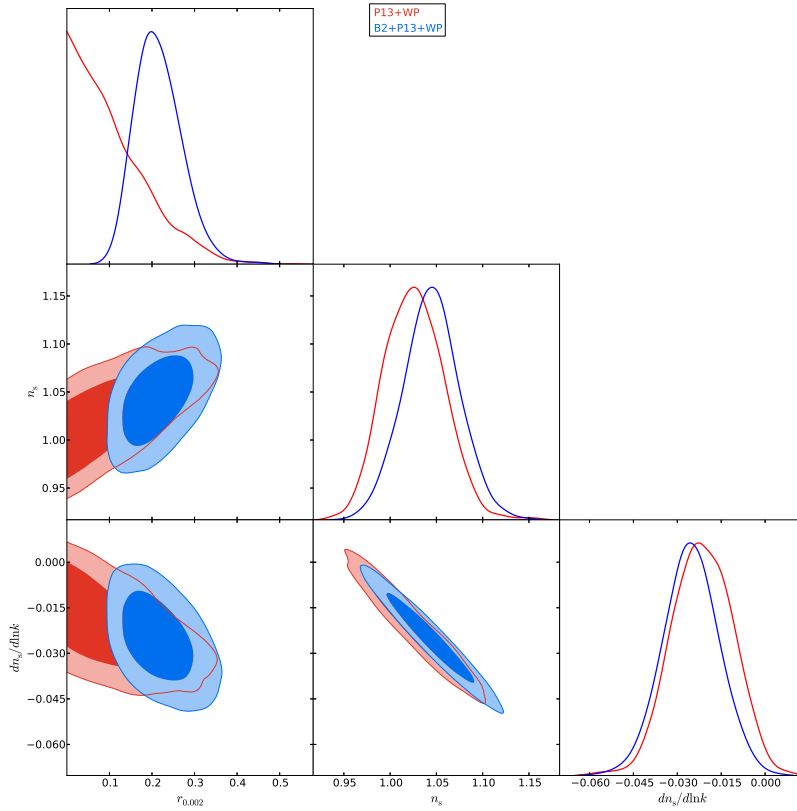


FIG. 2: The constraint contours on r , n_s and $dn_s/d\ln k$ from the combinations of P13+WP and B2+P13+WP in the Λ CDM+nrn+r model.

cates that this further parameter extension is favored at more than 1σ level. From Table III, we see that at the pivot scale $k_p = 0.002 \text{ Mpc}^{-1}$ the spectral index $n_s > 1$ is preferred at 2.2σ level, a negative running of spectral index is preferred at 2.2σ level and a positive running of running is preferred at 1.7σ level once the running of running is considered. Our results imply that higher order expansions might be considered in the future as well.

III. SUMMARY AND DISCUSSION

Discovery of relic gravitational waves opens a new window to explore cosmology. There are many possible sources for the relic gravitational waves, such as inflation [18–20], cosmic string [28, 29] and so on. In this paper we extend the Λ CDM+r cosmology to w CDM+r model and Λ CDM+r model with running spectral index, and find that the tension between B2 and P13 can be reconciled if a running spectral index is taken into account, but relaxing dark energy model does not work.

Usually inflation model predicts $|n_s - 1| \lesssim \mathcal{O}(10^{-2})$, $|dn_s/d\ln k| \lesssim \mathcal{O}(10^{-3})$ and $|d^2n_s/d\ln k^2| \lesssim \mathcal{O}(10^{-4})$. Our results imply that the simple canonical single-field slow-roll inflation models are not compatible with the datasets and the physics in the early Universe should be

much more complicated than what we expect if all of B2, P13 and WP are trustable. After B2 released its data, many authors investigated inflation models widely. See, for example, [30–40]. However almost all of them only tried to fit the value of tensor-to-scalar ratio and the spectral index. We believe that it is not enough because the combination of B2+P13+WP strongly implies a running spectral index. How to naturally achieve a significantly running spectral index is still an open question. As we known, the space-time non-commutative inflation [41–43] can generate a large negative running of spectral index. It can also be realized in the inflation with modulations [44–46] as well. Another possibility is that the Planck data is not reliable at all. In [47] we combine B2 with WMAP 9-year data and find that the power-law spectrum of scalar perturbation is compatible with B2+WMAP, and the power-law inflation and inflation model with inverse power-law potential can fit the data nicely. In a word, we believe that the realistic inflation model is still unknown and further investigation is needed in the near future.

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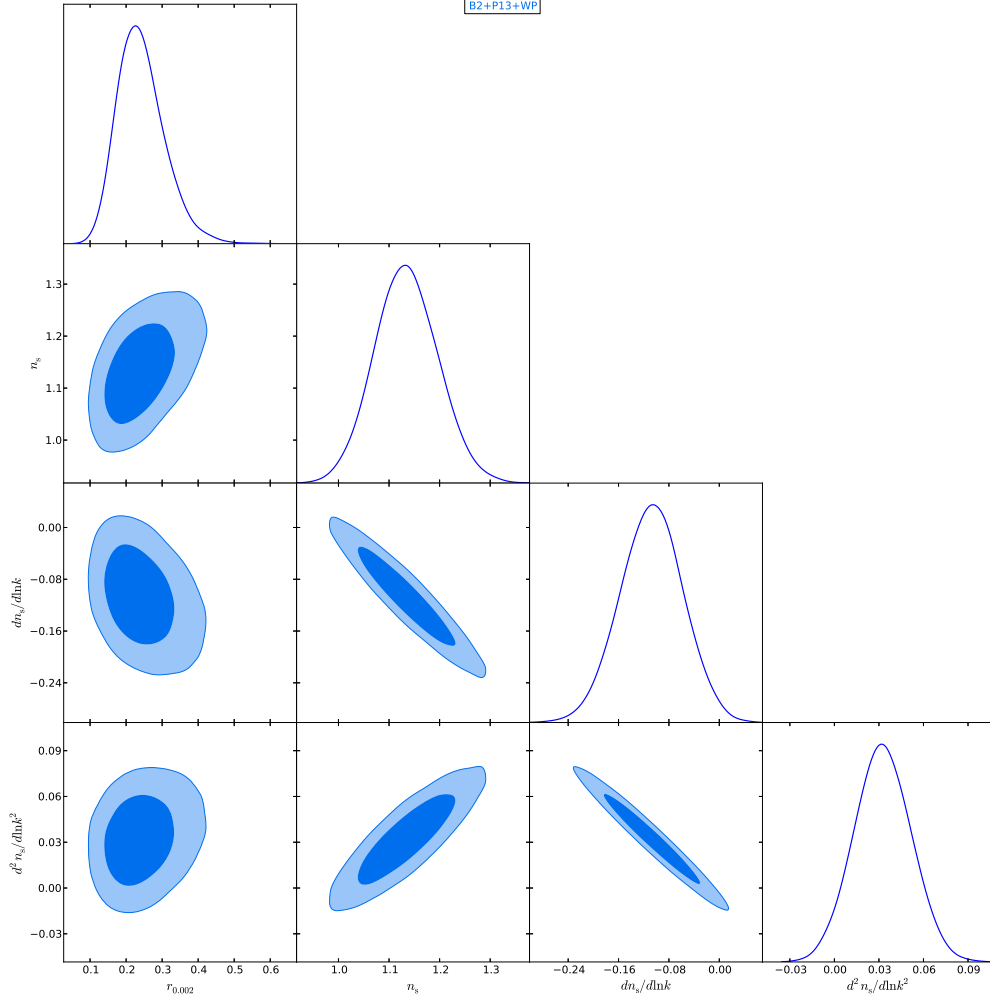


FIG. 3: The constraint contours on r , n_s , $dn_s/d\ln k$ and $d^2 n_s/d\ln k^2$ from the combination of B2+P13+WP in the Λ CDM+nrun+nrunrun+r model.

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